Evaluation of Shielding Effectiveness in the Time Domain using a DG Method with an Efficient PML

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The DG method in the time domain
- Heterogeneous media naturally included in the discrete formulation
- Easy parallelization for the numerical resolution
- Resolution in the time domain allows considering transient signals

PML for curved borders
- Perfectly matched for all wave incidences
- The proposed formulation allows considering curved truncation boundaries
- The use of hyperbolic absorption functions avoids any tuning

Methods

Physical context: TM waves
\[
\frac{\partial\varepsilon}{\partial t} - \nabla \times h = s_e \quad \text{with} \quad e = [0, 0, \varepsilon_z]
\]
\[
\frac{\partial h}{\partial t} - \nabla \times e = s_h
\]

Domain truncation: 2D PML, with curved borders

Additional source terms in Maxwell’s equations
\[
s_e = -\varepsilon (\sigma_{\text{pml}} - \sigma) \varepsilon - \varepsilon \sigma_h e
\]
\[
s_h = -\mu (\sigma_{\text{pml}} - \sigma) \frac{h}{t} - \mu \sigma_h h
\]

with \(\sigma_{\text{pml}}(d) = (k^{-1} + d^{-1})^{-1} \int_0^d \sigma_{\text{pml}}(r) \, dr\)

Additional equation
\[
\frac{\partial q}{\partial t} - \nabla \times h = -\varepsilon \sigma_{\text{pml}} q
\]

with \(q = [0, 0, q_z]\) and \(\nabla \times = [n \times \nabla]\)

New parameter: absorption function \(\sigma_{\text{pml}}(d)\)

Numerical solution: Discontinuous Galerkin scheme

Weak form for the element \(e\)
\[
\int_{\Omega_e} \left[ \frac{\partial e}{\partial t} \mathbf{\hat{e}} - \mathbf{h} \cdot (\nabla \times \mathbf{e}) - \mathbf{s}_e \cdot \mathbf{e} \right] \, d\Omega_e - \int_{\Gamma_{\text{n}}} [\mathbf{n} \times \mathbf{h}]^{\text{num}} \cdot \mathbf{\hat{e}} \, d\Gamma_e = 0
\]
\[
\int_{\Omega_e} \left[ \frac{\partial h}{\partial t} \mathbf{\hat{h}} + \mathbf{e} \cdot (\nabla \times \mathbf{h}) - \mathbf{s}_h \cdot \mathbf{h} \right] \, d\Omega_e + \int_{\Gamma_{\text{n}}} [\mathbf{n} \times \mathbf{e}]^{\text{num}} \cdot \mathbf{\hat{h}} \, d\Gamma_e = 0
\]
\[
\int_{\Omega_e} \left[ \frac{\partial q}{\partial t} \mathbf{\hat{q}} - \mathbf{h} \cdot (\nabla \times \mathbf{q}) + \frac{1}{\kappa^{-1} + d} (\mathbf{n} \times \mathbf{h}) \cdot \mathbf{\hat{q}} + \varepsilon \sigma_{\text{pml}} q \, \mathbf{\hat{q}} \right] \, d\Omega_e + \int_{\Gamma_{\text{n}}} [\mathbf{n} \times \mathbf{n}]^{\text{num}} \cdot \mathbf{\hat{q}} \, d\Gamma_e = 0
\]

Interface terms (upwind scheme)
\[
[n \times \mathbf{h}]^{\text{num}} = \frac{\mathbf{n} \times \mathbf{h}}{\mathbf{n} \times \mathbf{h}} \left( \frac{Z}{Z} \right) \left( \mathbf{n} \times \mathbf{n} \right) \, (\text{mean})
\]
\[
[n \times \mathbf{e}]^{\text{num}} = \frac{\mathbf{n} \times \mathbf{e}}{\mathbf{n} \times \mathbf{e}} \left( \frac{Z}{Z} \right) \left( \mathbf{n} \times \mathbf{n} \right) \, (\text{jump})
\]

Notation
- \(n, t\): Normal and tangent of the truncation boundary
- \(k\): Curvature of the truncation boundary
- \(d\): Distance between \(n\) and the truncation boundary
- \(\delta\): Thickness of the PML
- \(\mathbf{n}\): Surface of the element \(e\)
- \(\Gamma_{\text{e}}\): Boundary of the element \(e\)
- \(\mathbf{n}\): External normal of \(\Gamma_{\text{e}}\)

Numerical results

Scattering of a cavity
Different absorptions functions
\[
\sigma_1(d) = \alpha (d) / d^2 \quad \sigma_2(d) = \alpha (d) - \alpha (\delta - d)
\]
Relative mean error
\[
\frac{\int_0^\delta \int_0^1 \left[ \sigma_1 (d) - \sigma_2 (d) \right] \, dt \, dt}{\int_0^\delta \int_0^1 \left[ \sigma_1 (d) + \sigma_2 (d) \right] \, dt \, dt}
\]

Hyperbolic absorption functions \(\alpha\) and \(\alpha_0\) are nearly optimum with \(\alpha_0 = 2\alpha\).
The minimum error is of the same order for all absorption functions.

Hyperbolic functions are efficient and avoid any tuning.

Evaluation of the shielding effectiveness (long duration simulation)

Composite shielding

Gaussian incident signal
\[
E_{\text{inc}} = E_0 \exp(i(x - ct))/\sigma^2
\]

Shielding effectiveness
\[
S_{\text{eff}} = 20 \log_{10} \left( \frac{E_0}{E_{\text{eff}}} \right)
\]

Transverse magnetic case (5 inclusions)

Matrix: \(\varepsilon = 2, \mu = 1, \sigma = 0\) - Inclusion rate: \(\tau = 20%\)

Transverse electric case (15 inclusions)

\(\rightarrow\) Effective conductivity of the composite sheet: \(\sigma_{\text{eff}} = \tau \sigma_{\text{inc}}\)

\(\rightarrow\) Classical formula \(\sigma_{\text{eff}} = \tau \sigma_{\text{inc}}\) no longer valid.

Perspectives: comparison with other methods, other incident signals and 3D results

References