Open position for a 6-months internship at POEMS (UMR 7231) – 2018/2019

Preconditioned Boundary Element Methods for Time-Harmonic Wave Propagation

General context

The development of efficient approaches to simulate wave propagation in complex media is crucial for many topics *(e.g. in aeroacoustics, medical imaging, seismic risks, ...)*. It is still an open question due to the complexity of the system to model and the need of efficient tools for large-scale problems. Standard numerical approaches to simulate wave propagation range from domain methods as finite difference methods *(FDMs)* and finite element methods *(FEMs)* to boundary element methods *(BEMs)*.

The main advantage of the BEM is that only the domain boundary is meshed, leading to a drastic reduction of the number of degrees of freedom. The second advantage is that the radiation condition at the infinity is exactly integrated in the formulation for scattering problems. Nevertheless, 3D simulations are still limited to simplified configurations. Standard BEMs indeed lead to linear systems with a matrix that is both fully-populated and ill-conditioned, which severally limits the size of the problem, the geometric complexity and the frequency range that can be considered. This internship is part of an effort towards the development of tools to simulate real-life problems.

Scientific objective and methodology

Using iterative solvers *(e.g. GMRES)* for solving large-scale problems is attractive since they lead to highly parallelizable computational procedures. However, because the matrix of the system is ill-conditioned, the convergence of these solvers is very slow. Analytic preconditioning techniques [1,2] are investigated to accelerate the convergence. The approach relies on approximated Dirichlet-to-Neumann operators, which are also used as absorbing boundary conditions (ABCs) in the context of FEM solvers. Recent advances on these operators for geometric settings with corners [3] could be applied to accelerate the preconditioned BEM solvers and to extend the applicability of the approach. The objective of this internship is to propose, to implement and to analyze improved preconditioned BEM solvers for time-harmonic wave propagation.

First, the candidate will review and analyze existing preconditioning techniques for solving Helmholtz problems *(using Matlab codes for 2D numerical studies)*. Then, new preconditioners based on [3] will be investigated and tested for 2D settings of increasing complexity *(i.e. domains with a smooth border, with corners, non-convex, etc.)*. Depending on the results, the approach could be extended and tested on 3D problems using a FORTRAN code *(COFFEE, developed in the laboratory)* and the parallel performance could be investigated. Alternatively, the extension to elastic wave problems could be investigated.

Profile of the candidate

M2 student interested by the mathematical modelling and the numerical simulation. Good knowledge on wave-like PDEs, numerical methods and scientific programming. Basic knowledge on BEMs and parallel programming is an asset.

The internship will take place in the POEMS laboratory *(CNRS-ENSTA-Inria)*, within the Applied Mathematics Unit of ENSTA-ParisTech *(Palaiseau, France)*, under the co-supervision of Dr Stéphanie Chaillat and Dr Axel Modave, in collaboration with Dr Marion Darbas *(Université de Picardie Jules Verne)*.

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References

[1] Chaillat S, Darbas M, Le Louër F (2017). *Fast iterative boundary element methods for high-frequency scattering problems in 3D elastodynamics.* Journal of Computational Physics, 341, 429-446 [preprint]

[2] Darbas M, Darrigrand E, Lafranche Y (2013). *Combining analytic preconditioner and fast multipole method for the 3-D Helmholtz equation.* Journal of Computational Physics, 236, 289-316 [preprint]

[3] Modave A, Geuzaine C, Antoine X (20XX). *Corner treatment for high-order local absorbing boundary conditions in high-frequency acoustic scattering.* To submit. 27 pages.